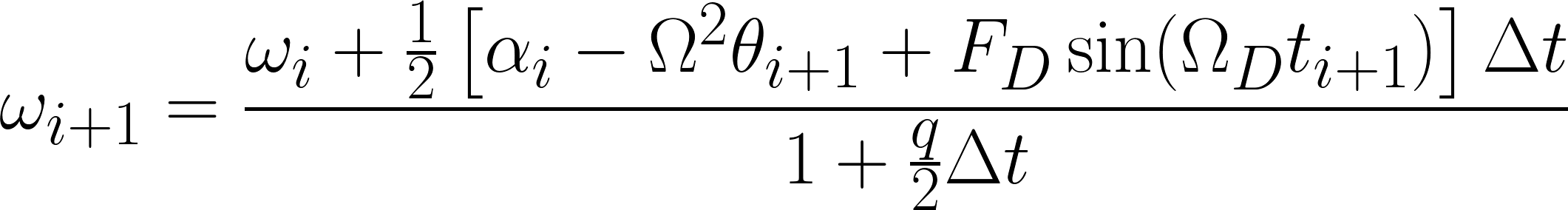
Computational Physics HW4b: Damped Driven Pendulums and Nonlinear Pendulums

1. Show that the leapfrog algorithm for computing the evolution in angular velocity is given by the following equation:



1. Write a program to solve the damped, driven pendulum using the leapfrog algorithm. You may assume an initial condition of angular velocity is zero but be prepared to do the general case in HW 4c.
2. Show evidence that the numerical solution to the underdamped non-driven pendulum matches the analytical solution.
3. Show evidence that the numerical solution to the critically damped non-driven pendulum matches the analytical solution.
4. Show evidence that the numerical solution to the over damped non-driven pendulum matches the analytical solution.
5. Construct plots for angular position vs. time for each of the three cases.
6. Construct phase space diagrams for each of the three cases (what should go on the y-axis in this case?).
7. What conclusions can be drawn about the energy of the pendulum in each of the three cases? What other conclusions can you make? Discuss.
   1. In the underdamped case, energy is lost slowly so that the pendulum has kinetic energy left over when it reaches the zero angular position and converts it back into potential energy.
   2. In the critically damped case, energy is lost at a rate so that when the pendulum reaches the zero angular position, it has lost all kinetic energy.
   3. In the overdamped case, energy is lost so quickly that the potential energy is not converted into kinetic energy quickly enough to be lost at the fastest rate because damping depends on velocity, and velocity depends on kinetic energy.
8. Now include a driving force.
9. For each of the three cases construct: plot of angular position vs. time and phase space diagram.
10. For each of the three cases construct a plot of amplitude vs. driving angular frequency/natural frequency. Where does the maximum occur?
    1. Maximum occurs when natural frequency = driving frequency (resonance)
11. In the plots constructed in #10, what effect if any does q have on the peak and width of the curve? Explain why using physics.
    1. Lower q values lead to higher amplitudes because the damping does not affect the motion as much. A lower q value also decreases the width of the curve (wavelength?) because the motion not as slowed by damping. (This correlation is showed by the graph of 11 not 10, but the ones in 10 could be used to show it.)
12. ~~For each of the three cases construct a plot of phase lag (the difference in phase between the angular position and the driving force) vs. amplitude. What conclusions can be drawn? Explain.~~
13. Write a program to solve the nonlinear pendulum with no damping and no driving force using the leapfrog algorithm.
14. Construct a plot of angular position vs. time graph using a small angular displacement as the initial condition. Compare the results to the linearized (SHO) case. Explain your findings.
    1. At small angles, so the results should be almost identical.
15. Construct a plot of angular position vs. time graph using a large angular displacement as the initial condition. Compare the results to the linearized (SHO) case. Explain your findings.
    1. At large angles near π, so the nonlinear pendulum does not change direction as quickly as the linearized pendulum. As a result, the two graphs are different in period.
16. For large initial angular displacement, construct a plot of period vs. amplitude. What should the plot be in the linearized case? Compare the two and explain how they are different and why.
    1. In the linearized case, the period would be consistent for any amplitude, but in the nonlinear case, the period is larger for larger angles. This is because in the nonlinear case, the acceleration is related to which is smaller at large angles and the pendulum spends more time in at those large angles.